

Mathematical model with autoregressive process for electrocardiogram signals

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Abstract

The cardiovascular system is composed of the heart, blood and blood vessels. Regarding the heart, cardiac conditions are determined by the electrocardiogram, that is a noninvasive medical procedure. In this work, we propose autoregressive process in a mathematical model based on coupled differential equations in order to model electrocardiogram signals. Our results are compared with experimental tachogram by means of Poincaré plot and detrended fluctuation analysis. We verify that the results from the model with autoregressive process show good agreement with experimental measures from tachogram generated by electrical activity of the heartbeat. With the tachogram we build the electrocardiogram by means of coupled differential equations.

Keywords: Heartbeat, Autoregressive, Electrocardiogram

1. Introduction

The cardiovascular system (CVS) is responsible for supplying the human organs with blood. It is composed by the heart, the arteries, and the veins. The heart has as function to pump blood throughout the body, that is realised by means of contractions [1]. The human heart beats an average 72 beats per minute and pumps 0.07 liters of blood per beat [2, 3]. The contraction and relaxation of the heart is obtained by a single cycle of the electrocardiogram signal (ECG), namely the ECG records the electrical activity of the heart. Waller in 1887 [4] measured for the first time the electrical activity from the heart, and the first practical electrocardiograph was invented by Einthoven in 1901 [5] that it was used as a tool for the diagnosis of cardiac abnormalities.

In the recent past, several theoretical investigations pertaining to CVS have been carried out to analyse electrocardiogram signal [6, 7, 8]. Mathematical models have been developed to understand physiological function and dysfunction in CVS. A mathematical model which have been used to generate ECG signals is the Van de Pol oscillator [9]. Gois and Savi considered three modified Van der Pol oscillators connected by time delay coupling to describe heart rhythm behaviour [10]. The coupled Van der Pol oscillators was also used in studies about the control of irregular behaviour in pathological heart rhythms [11]. McSharry and collaborators [12] introduced a dynamical model to describe generating synthetic electrocardiogram signals. This model is based on a set of three ordinary differential equations in that it is incorporated the respiratory sinus arrhythmia (RSA) by means of a bimodal power spectrum consisting of the sum of two Gaussian distributions.

In this work, instead two Gaussian distributions we propose an autoregressive (AR) process for the RSA to obtain the ECG. Boardman and collaborators [13] study autoregressive model for the heart rate variability (HRV) [14, 15]. They found the optimum order of autoregressive model which can be used for spectral analysis of short segments of tachograms. We compare the results obtained from two Gaussian distributions and AR with experimental tachogram. To do that, we use as diagnostic tools the Poincaré plot [16] and detrended fluctuation analysis (DFA) [17]. We have verified that the result with AR process agrees with the experimental tachogram more closely than the result with two Gaussian distributions. This way, we generate the ECG by means of coupled differential equations considering the AR process to obtain the angular velocity of the tachogram. The angular velocity is an important parameter in the mathematical model for the ECG

signal, and variation in the angular velocity produces variation in the times elapsed between successive heartbeats.

This article is organised as follows: in Section 2 we introduce the model with autoregressive process for tachograms, in Section 3 we compare our results with experimental datas, and in the last Section we draw our conclusions.

2. The mathematical model

The ECG is a noninvasive method used to measure electrical activity of the heart through electrodes placed on the surface of the skin. Figure 1 shows the relationship between the cardiac conduction and the ECG. In the sinoatrial node (SA), known as natural pacemaker, the heartbeat starts. The atrioventricular node (AV) is responsible for the passage of electrical signals from the atria to the ventricles. At last, the signal arrives at the Purkinje fibers and makes the heart contract to pump blood, where the R-peak occurs. The time between successive R-peaks is the RR-interval and the series of RR-intervals is known as RR tachogram.

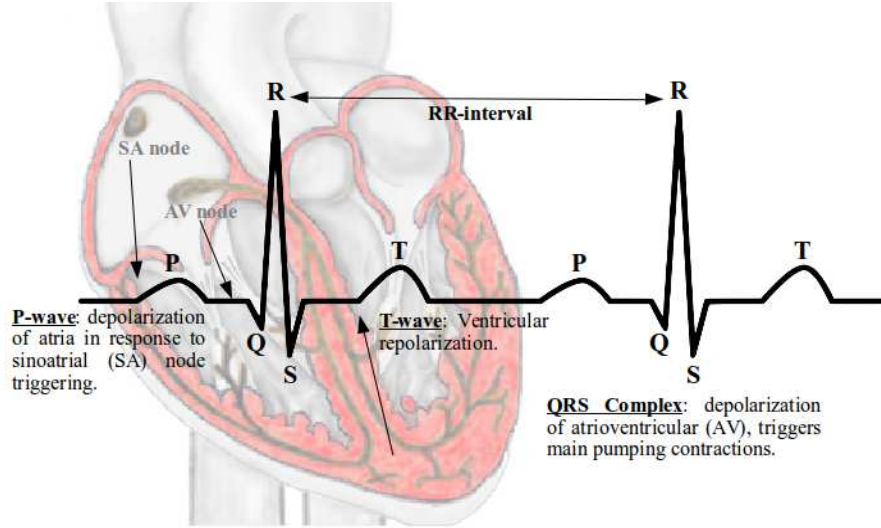


Figure 1: (Colour online) ECG following a normal heartbeat.

MacSharry and collaborators [12] argued that the heartbeat can be described by means of three coupled ordinary differential equations with the

Table 1: Parameters for Equation (1).

index (i)	time (s)	θ_i	a_i	b_i
P	-0.2	$-\pi/3$	1.2	0.25
Q	-0.05	$-\pi/12$	-5.0	0.1
R	0	0	30.0	0.1
S	0.05	$\pi/12$	-7.5	0.1
T	0.3	$\pi/2$	0.75	0.4

inclusion of RSA at the high frequencies (f_{RSA}) and Mayer waves (MW) at the low frequencies (f_{MW}). The equations are given by

$$\begin{aligned}\dot{x} &= \alpha x - \omega y, \\ \dot{y} &= \alpha y + \omega x, \\ \dot{z} &= z_0 - z - \sum_i a_i \Delta\theta_i e^{-\frac{\Delta\theta_i^2}{2b_i^2}},\end{aligned}\tag{1}$$

where $i \in \{P, Q, R, S, T\}$, $\alpha = 1 - \sqrt{x^2 + y^2}$, $\Delta\theta_i = \theta - \theta_i \pmod{2\pi}$, $\theta = \text{atan2}(y, x)$ ($-\pi \leq \text{atan2}(y, x) \leq \pi$), $z_0(t) = A \sin(2\pi f_{\text{RSA}} t)$, and $A = 0.15\text{mV}$. The other parameters are given in Table 1 according to Reference [12].

The angular velocity ω controls the variations in the RR-intervals, and it is given by

$$\omega(t) = \frac{2\pi}{T(t)},\tag{2}$$

where $T(t)$ is the RR-interval time series obtained from the inverse Fourier transform of the sum of two Gaussian distributions [12]

$$|H_G(f)| = \frac{\sigma_{\text{MW}}^2}{\sqrt{2\pi c_{\text{MW}}^2}} \exp \frac{(f - f_{\text{MW}})^2}{2c_{\text{MW}}^2} + \frac{\sigma_{\text{RSA}}^2}{\sqrt{2\pi c_{\text{RSA}}^2}} \exp \frac{(f - f_{\text{RSA}})^2}{2c_{\text{RSA}}^2}.\tag{3}$$

Figure 2(a) exhibits the power spectrum $|H_G(f)|$ for $f_{\text{MW}} = 0.1\text{Hz}$, $f_{\text{RSA}} = 0.25\text{Hz}$, $c_{\text{MW}} = 0.01$, $c_{\text{RSA}} = 0.01$, and $\sigma_{\text{MW}}^2/\sigma_{\text{RSA}}^2 = 0.5$. The spectrum has a bimodal form, where one peak is located in the low frequency range $0.04 \leq f < 0.15\text{Hz}$ and the other is located in the high frequency range $0.15 \leq f \leq 0.4\text{Hz}$. These two bands appear due to the effects of both Mayer waves and RSA. The tachogram is generated by the inverse Fourier transform from the power spectrum $|H_G(f)|$. The time interval between two

Table 2: Coefficients values for the AR power spectrum density (Eq. 5).

$d_1 = -0.9099$	$d_2 = 0.5188$	$d_3 = -0.2840$	$d_4 = -0.2063$
$d_5 = 0.0382$	$d_6 = 0.0709$	$d_7 = 0.0305$	$d_8 = -0.1533$
$d_9 = 0.0009$	$d_{10} = -0.0070$	$d_{11} = -0.0218$	$d_{12} = 0.0043$
$d_{13} = 0.0316$	$d_{14} = 0.0155$	$d_{15} = -0.0591$	$d_{16} = 0.0252$

consecutive R-peaks is denoted by $r(n)$ ($0 \leq n \leq N - 1$), as shown in Figure 1. With the sequence of the $r(n)$ values (tachogram) is possible to calculate the angular velocity $\omega(t)$, and to build the ECG by means of Equation 2.

In this work, in order to model electrocardiogram signals we propose an autoregressive (AR) process [13] to determine $\omega(t)$. The AR process of order p is defined as

$$R(n) = \sum_{l=1}^p d_l R(n-l) + \epsilon(n), \quad (4)$$

where $\epsilon(n)$ is a white noise with zero mean and unit variance. Boardman and collaborators [13] found that $p = 16$ is an optimum order of autoregressive model for heart rate variability. The AR power spectrum density is

$$|H_{AR}(f)| = \frac{1}{|1 - \sum_{l=1}^p d_l e^{-i2\pi fl}|}, \quad (5)$$

with the coefficients values given in Table 2

Figure 2(b) shows the power spectrum calculated from the tachogram generated by means of the AR process. In placed of the two separated Gaussian distributions, the AR process produces a damped in the power spectrum, and as consequence the separation between the frequency components cannot be exactly identified. This way, with the tachogram we find $\omega(t)$ and build the ECG using Equation (1).

3. Results and discussions

In this work, the power spectrum is considered to obtain the theoretical tachogram and consequently the angular velocity (Eq. 2) that is used in Equation (1) to build the ECG. We calculate the power spectrum from Gaussian distributions (blue), AR process (red), and experimental data (green), as shown in Figure 3(a).

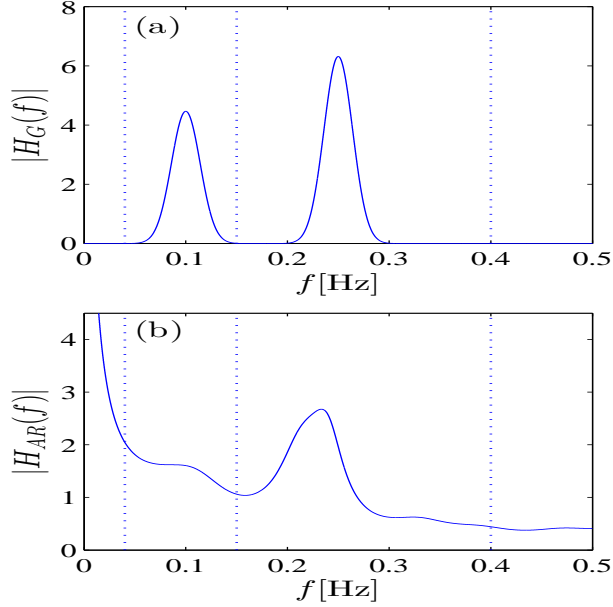


Figure 2: Power spectrum from (a) Equation (3) and (b) Equation (5).

With regard to Figure 3(a), we see that the power spectrum from AR process has a shape closer the experimental result than the power spectrum from the Gaussian distributions. This way, in order to verify the agreement among the experimental tachogram and the tachograms obtained from theoretical power spectra we have utilised the detrended fluctuation analysis (DFA) [18, 19]. DFA yields a fluctuation function $F(k)$ as a function of k , given by [17]

$$F(k) = \sqrt{\frac{1}{N} \sum_{n=1}^N [r(n) - r_k(n)]^2}, \quad (6)$$

where $N = 1000$, k is the box size that partitions the time interval of the tachogram, and $r_k(n)$ is the local trend in each box. DFA is a nonlinear dynamical analysis that have been used for the understanding of biological systems [17]. Moreover, the DFA allows the detection of long-range correlations embedded in a patchy landscape.

Figure 3(b) shows the DFAs for $0 < k < 100$, where linear regressions present slopes equal to 0.1955 ± 0.0150 , 0.7034 ± 0.0686 , and 0.6817 ± 0.2448 for Gaussian distributions, AR process and experimental data, respectively.

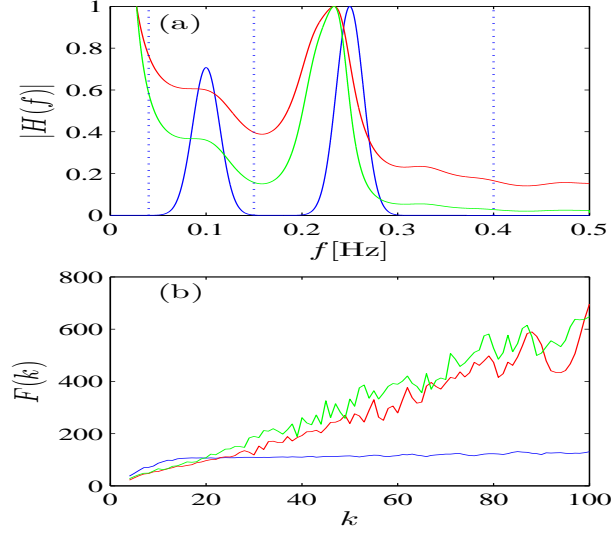


Figure 3: (Colour online) (a) Power spectrum and (b) detrended fluctuation analysis (DFA) for Gaussian distributions (blue), AR process (red), and experimental data (green).

The experimental data are collected from 62 individuals. As a result, the DFA for the tachogram generated by the experimental data and AR process are in close agreement with each other. Whereas DFA for the Gaussian distributions exhibits a good agreement only for $k < 20$.

Through the inverse Fourier transform from the two Gaussian distribution with phases randomly distributed between 0 and 2π we build the tachogram, shown in Figure 4(a). The tachogram generated by AR process is illustrated in Figure 4(b) and the experimental tachogram is in Figure 4(c). In Figures 4(d), 4(e), and 4(f) we calculate the respective Poincaré plots. The Poincaré plot is a visualising technique to analyse RR intervals, where it is computed the standard deviation of points perpendicular to the axis (SD1) and points along (SD2) the axis of line-of-identity. Table 3 exhibits the SD1 and SD2 values of the tachograms shown in Figure 4. Comparing the Poincaré plots we see that both SD1 and SD2 for the AR process agree with the experimental results better than the method based on the Gaussian distribution.

All in all, we calculate the angular velocity from the tachogram generated by the AR process. Then, we use the angular velocity in Equation (1) to obtain the ECG signal. Figure 5 shows the ECG signal in the time interval $0 \leq t \leq 20$ s, where $z(t)$ yields a synthetic ECG with a realistic PQRST

Table 3: SD1 and SD2 values for the tachograms shown in Figure 4.

RR-intervals	SD1	SD2	
Gaussian distribution	63.9129 ± 1.9857	103.4129 ± 2.5682	Fig. 4(d)
AR process	38.2919 ± 2.6989	105.6210 ± 3.9716	Fig. 4(e)
Experimental data	38.8613 ± 19.6376	79.0452 ± 28.5804	Fig. 4(f)

morphology according to Figure 1.

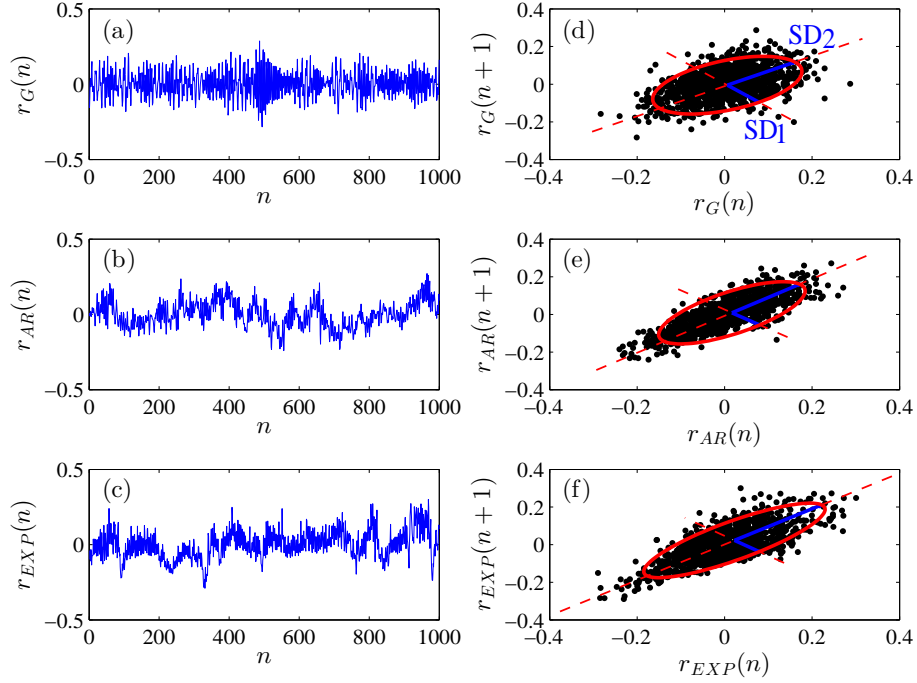


Figure 4: (Colour online) Tachograms generated by (a) the sum of two Gaussian distributions, (b) the AR process, (c) experimental data, and respective Poincaré plots in (d), (e), and (f).

4. Conclusions

In conclusion, we have studied a mathematical model given by coupled differential equations that describes electrocardiogram signals. In the original model is considered a power spectrum with two Gaussian distributions to

build the tachogram. Through the tachogram is obtained the angular velocity that is used in the original mode. In this work, we propose to calculate the angular velocity by means of the AR process.

We verify that the power spectrum from AR process has a good agreement with the power spectrum from experimental data. We have also compared the tachograms generated from Gaussian distributions and AR process with the experimental tachogram using DFA and Poincaré plot. As a result, in both DFA and Poincaré plot, the tachogram generated considering the AR process is in closer agreement with experimental than the two Gaussian. As a consequence, with the tachogram, the angular velocity is calculated and the ECG signal can be built utilising coupled differential equations with AR process.

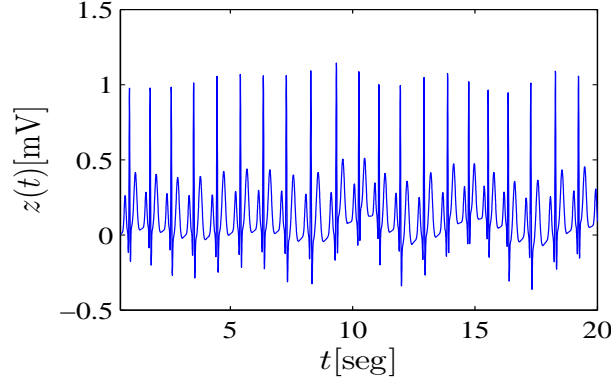


Figure 5: ECG signal generated by means of Equation (1) considering AR process.

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